Homework #2, PHY 674, 1 September 1995

(X6). Consider the equilateral triangle group C_{3v} (Joshua, page 7). Find all subgroups of this group. Which of the subgroups are Abelian ? Which subgroups are normal subgroups ? (4 points)

(X7). Find all conjugacy classes of the equilateral triangle group and determine the multiplication table for the multiplication of classes (see Joshua, Section 1.4.5, or Tinkham, Section 2-9). Now interpret this result geometrically: What kind of symmetry operations are conjugate to each other ? What do conjugate matrices have in common ? (4 points)

(X8). Consider the following map:

$$f : \mathbb{R} \to \mathbb{R}^+ \tag{8.1}$$
$$t \mapsto f(t) = \exp(t) \tag{8.2}$$

Given the additive group structure of $(\mathbb{R}, +)$ and the multiplicative group structure of $(\mathbb{R}^+, \cdot)$, show that f is a homomorphism of groups. Find the image and the kernel of f. Is the map injective, surjective, bijective ? If it is bijective, find the inverse map. If it is injective (and not surjective), restrict the map to its image

$$f : \mathbb{R} \to f(\mathbb{R}) \subseteq \mathbb{R}^+ \tag{8.3}$$

and then find the inverse map of this restriction. (4 points)

(X9). Prove Tinkham's **Rearrangement Theorem**, see Section 2-3: In the multiplication table of a (finite) group, every element appears only once in each row. (2 points)

(X10). Write down the symmetry operations of the square (in the plane) using the Schoenflies notation for symmetry elements. Write down its multiplication table. This group is called C_{4v}. (4 points)

(X11). Show that there is a non-trivial homomorphism from the cyclic group order 4 to the cyclic group of order 2. What are the image and kernel of this homomorphism ? (4 points)

Due Date:
Friday, 8 September 1993, 2:10 pm,

in class or in the green homework box just inside the south entrance to Room 12.

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Send questions to: `zollner@iastate.edu`.